

# Dividing Polynomials

## Long Division

Recall:

divisor  $\rightarrow$  12 =

$$\begin{array}{r}
 545 \overline{) 6543} \\
 \underline{-60} \phantom{0} \phantom{0} \phantom{0} \\
 54 \phantom{0} \phantom{0} \phantom{0} \\
 \underline{-48} \phantom{0} \phantom{0} \phantom{0} \\
 63 \phantom{0} \phantom{0} \phantom{0} \\
 \underline{-60} \phantom{0} \phantom{0} \phantom{0} \\
 3 \phantom{0} \phantom{0} \phantom{0}
 \end{array}$$

$\frac{3}{12}$   
 $\rightarrow$  dividend

Quotient + remainder

$$545 + \frac{3}{12}$$

$\uparrow$   
divisor

Ex: ① Divide  $x^3 + 2x^2 - x - 2$  by  $x + 1$

$$\begin{array}{r}
 \textcircled{x+1} \overline{) \textcircled{x^3} + 2x^2 - x - 2} \\
 \underline{-(x^3 + 1x^2)} \phantom{-x - 2} \\
 \phantom{x^3} + x^2 - x - 2 \\
 \underline{-(x^2 + 1x)} \phantom{-2} \\
 \phantom{x^3} \phantom{x^2} - 2x - 2 \\
 \underline{-(-2x - 2)} \\
 \phantom{x^3} \phantom{x^2} \phantom{-2x} 0
 \end{array}$$

\* Divide 1st term by 1st term\*

Quotient:  
 $x^2 + x - 2$

Since there is no remainder,  $x+1$  is a factor of  $x^3 + 2x^2 - x - 2$

② Divide  $x^3 - 4x^2 + x + 6$  by  $x - 5$

$$\begin{array}{r}
 x^2 + x + 6 \\
 x - 5 \overline{) x^3 - 4x^2 + x + 6} \\
 \underline{-(x^3 - 5x^2)} \phantom{+ x + 6} \\
 \phantom{x^3 - } 11x^2 + x + 6 \\
 \phantom{x^3 - } \underline{-(11x^2 - 55x)} \phantom{+ 6} \\
 \phantom{x^3 - } \phantom{11x^2 + } 56x + 6 \\
 \phantom{x^3 - } \phantom{11x^2 + } \underline{-(56x - 280)} \\
 \phantom{x^3 - } \phantom{11x^2 + } \phantom{56x + } 286
 \end{array}$$

$x^2 + x + 6 + \frac{36}{x-5}$

36 → remainder

③ Divide  $2x^4 + 3x^3 + 5x - 1$  by  $x^2 + 3x + 2$

Missing  $x^2$  term,  
You must hold its place  
with  $0x^2$

$$\begin{array}{r}
 2x^2 - 3x + 5 \\
 x^2 + 3x + 2 \overline{) 2x^4 + 3x^3 + 0x^2 + 5x - 1} \\
 \underline{-(2x^4 + 6x^3 + 4x^2)} \phantom{- 1} \\
 \phantom{2x^4 + } -3x^3 - 4x^2 + 5x \phantom{- 1} \\
 \phantom{2x^4 + } \underline{-( -3x^3 - 9x^2 - 6x)} \phantom{- 1} \\
 \phantom{2x^4 + } \phantom{-3x^3 - } 5x^2 + 11x - 1
 \end{array}$$

Q +  $\frac{R}{\text{divisor}}$

$$2x^2 - 3x + 5 + \frac{-4x - 1}{x^2 + 3x + 2}$$

$$\begin{array}{r}
 -3x^3 - 4x^2 + 5x \\
 \underline{-( -3x^3 - 9x^2 - 6x)} \\
 5x^2 + 11x - 1 \\
 \underline{-(5x^2 + 15x + 10)} \\
 -4x - 11 \uparrow \\
 \text{remainder}
 \end{array}$$

④ Divide  $x^4 + 2x^2 - x + 5$  by  $x^2 - x + 1$   
 $+ 10x^3$

$$\begin{array}{r}
 x^2 + x + 2 \\
 \hline
 x^2 - x + 1 \overline{) x^4 + 0x^3 + 2x^2 - x + 5} \\
 - (x^4 - x^3 + x^2) \phantom{- x + 5} \\
 \hline
 x^3 + x^2 - x \phantom{+ 5} \\
 - (x^3 - x^2 + x) \phantom{+ 5} \\
 \hline
 2x^2 - 2x + 5 \\
 - (2x^2 - 2x + 2) \\
 \hline
 3
 \end{array}$$

$$\boxed{x^2 + x + 2 + \frac{3}{x^2 - x + 1}}$$